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ON SLIP AND YIELDING OF ALLOYS

WITH LAMELLAR MICROSTRUCTURES

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Abstract

The flow behavior of an alloy with a two-phase lamellar microstructure
is analyzed. Based on the model presented, the macroscopic slip system
and the yield strength are controlled by the ability of a slip system,
once activated in the softer phase, to shear the plate-like harder phase.
This ability to shear is controlled by the capacity of a specific slip
system, together with the applied stress, to generate sufficient shear
stress through the thickness of the harder phase to cause macroscopic flow.
The yield stress and active macroscopic slip plane of an individual
Widmanstatten colony of α-β Ti alloy are analyzed to demonstrate the
model. Comments are also made regarding the deformation of and crack
formation in pearlitic steels in terms of the proposed model.

INTRODUCTION

While the concepts of particle hardening have received considerable

attention, relatively little is known about the deformation of alloys in

which the strengthening phase possesses a lamellar morphology. Such micro-

structures are quite common, and include: β-annealed α(hcp)-β(bcc) Ti alloys,

pearlitic steels, and certain metal-matrix composites. At least in the

α-β Ti alloys, the flow behavior of individual colonies of such a micro-

structure is very dependent on colony orientation with the yield stress not

obeying any obvious form of Schmid's law.[1] The observed behavior cannot be

rationalized in terms of existing concepts of particle hardening. The

purpose of this communication is to present a straightforward model to

account for yielding of individual colonies of an alloy with an aligned

plate-like or lamellar microstructure. The yield stress and active macro-

scopic slip plane of a Ti-8Al-1Mo-1V Widmanstatten colony will be examined

as an application of the model. Some comments regarding the deformation of

and crack formation in pearlite will also be made.

THE MODEL

A schematic of the flow model is illustrated in Fig. 1. The essential

idea is that macroscopic flow in an aligned two phase alloy occurs as a result

of a combination of the external stress σ and an internal shear stress τ caused

by slip in the softer phase. Conceptually, the model is similar to that of

Courtney for the fracture of composites.[2] Slip is assumed to initiate in the

α-phase and impinges on the plate-like β phase. Elastic strains in the β are

caused both by the axial stress and the impinging slip band. Thus, an accurate

description of the stress state in the β phase must include the effects of the

axial stress as well as the impinging slip.

In order for macroscopic yielding to occur, the maximum shear stress in the β-phase, τ_{max}, must equal the critical resolved shear stress of the β*, if the β phase is ductile. Transforming the shear stress τ due to slip in the α plus the applied compressive stress σ into the coordinate system of the β phase (see Fig. 1), we obtain:

$$\tau_{max} = \left[\left(\frac{\sigma}{2} - \sigma\cos^2\beta_s + \tau\sin2\beta_s\right)^2 + \left(\frac{\sigma}{2}\sin2\beta_{sa} - \tau\cos2\beta_{sa}\right)^2\right]^{1/2}. \tag{1}$$

The angles β_s and β_{sa} are shown in Fig. 1 and are resp., the angles between the normal to a β plate and the stress axis (β_{sa}) or the normal to slip plane (β_s). If macroscopic yielding is controlled by fracture of the β phase, then a maximum normal stress criteria should apply instead of eq. 1, and this will be discussed subsequently in the section on pearlite deformation.

In the above relation, the internal shear stress component τ on the β develops only as α plastically deforms. However, owing to their morphology, the β plates can only shear through their thickness. Thus, τ is dependent on only to that component of the shear strain in the α which, while confined to the slip plane, will cause shear through β plate thickness. For a given α-phase slip band with shear strain γ_p and whose slip vector makes an angle β_b with the β plate normal, we thus have:

$$\tau = \mu \, \gamma_p \, \sin\beta_s\cos\beta_b, \tag{2}$$

where μ is the shear modulus of the β.

If slip in the α is uniform prior to the β plate yielding (in some cases, this may occur if it is the yielding of β which causes inhomogeneous slip,[1] then γ_p is related to the macroscopic axial plastic strain in the α-phase ε_p by the relation:[3]

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*The β-phase is assumed to possess sufficient multiplicity of slip planes so that a slip plane is near the orientation of the maximum shear stress plane.

$$\gamma_p = \frac{1}{\sin\theta_s} \{ ((1+\epsilon_p)^2 - \sin^2\phi_s)^{1/2} - \cos\phi_s) \}. \tag{3}$$

where θ_s and ϕ_s are the angles between the stress axis and the slip plane (θ_s) and the slip direction (ϕ_s) in the α. At small γ_p (< .05), eq. 3 may be simplified to:

$$\gamma_p \simeq \epsilon_p (\sin\theta \, \cos\phi)^{-1}. \tag{4}$$

Combining eqns. 2 and 4, τ depends on axial strain ϵ_p in the α under uniform slip conditions by:

$$\tau \cong \frac{3}{8} E\epsilon_p \, \sin\beta_s \cos\beta_b (\sin\theta_s \quad \cos\phi_s)^{-1}, \tag{5}$$

where E is the Young's modulus of the β phase. The axial stress σ can also be related to ϵ_p by noting that β responds to σ in an elastic manner prior to its yielding so that:

$$\sigma = E\epsilon_{TOT} = E\left(\frac{\tau_y^\alpha}{E_\alpha \cos\theta_s \cos\phi_s} + \epsilon_p\right), \tag{6}$$

where the total strain in the β, ϵ_{TOT}, is simply related to the critical resolved shear stress for a specific slip system in the α, τ_y^α, and the Young's modulus of the $\alpha \cdot$ phase. This assumes equal strain between the α and β phases, which is correct so long as $\beta_{sa} \neq 0$ and if slip is not parallel to the β lamellae.

Combining eqns. 1, 5, and 6, we are now able to calculate τ_{max} as a function of strain ϵ_p on a given slip plane in the α. Such a calculation will indicate the relative ability of a given slip system to shear through the β barrier. If τ_{max} increases relatively rapidly with plastic strain on a specific active α slip system, the "shearing ability" of this slip system is large and macroscopic slip may occur on this slip plane even though there may be a relatively small shear stress for slip on this system. Thus,

macroscopic slip on a system with a very small Schmid factor (with regard to the α-phase) is possible. Furthermore, if flow on those α-phase slip planes with a high shear stress cannot penetrate the β, a high axial yield stress will result because of the necessity for activating other slip systems with small Schmid factors but with greater ability to shear the β phase. Thus both the active macroscopic slip plane and the yield stress are controlled by the ability of a given slip system in α, once activated, to shear through the plate-like β barrier. It might be noted that for the case of an active slip system with a high Schmid factor ($\theta_s = \phi_s = 45°$), the maximum τ_{max} at a given ε_p occurs when the β plates are inclined to the stress axis at $\beta_{sa} \simeq 50°$.

The above model is obviously a simplified one, and we recognize that there are other factors which are important in the deformation of alloys with lamellar microstructures. However, we do feel that our model represents a critical aspect in the slip and yielding of alloys with lamellar microstructures. An obvious factor which has been ignored is the inhomogeneity of slip in the α-phase prior to macroscopic yielding. This will serve to intensify the τ term in eqn. 1. Thus if slip in the softer phase is quite inhomogeneous, it is likely that the first slip system activated will shear the harder phase, and yielding will occur at a low macroscopic yield stress and on a slip plane of high Schmid factor. Such behavior has important implications in the degree to which a given colony exhibits inhomogeneous flow, and this will be examined in some detail in a subsequent publication. Other factors, such as local stress distributions[4] or degree of cross slip and therefore stress relaxation at the α-β interface or the spacing between β plates should also be contained in the τ term of eqn. 1. However, specific models would be required to incorporate these factors into a model for flow.

APPLICATION OF THE MODEL

In order to demonstrate the application of the model, we present in Fig. 2 the τ_{max} vs. ε_p curves calculated, on the basis of uniform slip prior to yielding, for an individual Widmanstatten colony of the α-β Ti-8Al-1Mo-1V alloy. The limited slip systems available in the softer hcp α-phase matrix of this alloy serve to accentuate the effects on the slip and yielding behavior due to the plate-like barrier (which is martensitic α' in this heat treatment). Using eqns. 1, 4, and 5, the curves for τ_{max} in the β phase as a function of ε_p in the α-phase are calculated for each of five most highly stressed slip planes. Despite the fact that there is another prism plane which has a considerably higher shear stress (255 MPa) on it, the active macroscopic slip plane in the sample analyzed in Fig. 2 is the $(1\bar{1}00)$ prism plane with only a critical resolved shear stress of 193 MPa. Other samples tested in this heat treatment show that macroscopic flow for differing α-phase/β plate orientations can occur on the (0001) at 330 MPa and on the $\{10\bar{1}1\}$ at 241 MPa. Thus, in the sample analyzed in Fig. 2 there is sufficient applied stress to activate micro-slip on the (0001) and the $(0\bar{1}11)$ and possibly the $(\bar{1}011)$. However, as a result of its high ability to shear the barrier phase, the $(1\bar{1}00)$ plane is macroscopic slip plane even though there are at least three other slip systems (each with low penetrating ability) which should have been active prior to extensive slip on the $(1\bar{1}00)$. This concept has important implications in determining the inhomogeneity of flow in such micro-structures, and will be discussed in an analysis of the deformation behavior of Ti-8Al-1Mo-1V alloy colonies to be published later.[1]

COMMENTS ON THE DEFORMATION AND FRACTURE OF PEARLITE

The proposed model is useful in examining certain features regarding the deformation of and crack formation in pearlitic steels. Slip within pearlite

colonies occurs by shear both parallel to and across the cementite lamellae.[5,6] Several studies indicate that the cementite behaves in a brittle manner, at least in the tensile deformation of pearlite at room temperature.[7-10] When shear occurs across the cementite plates, cracking of the cementite appears to be slip-induced.[11-13] The cracked cementite plates subsequently lead to cracks across part of a pearlite colony.[11-13] Several investigators have noted the geometry of pearlite cracks,[11,13,14] and Miller and Smith show statistically that cracks which occur in pearlite colonies tend to lie at ≈50° to the tensile axis and tend to occur in colonies where the lamellae are aligned parallel to the stress axis.[12] Based on their observations, Miller and Smith suggest a mechanism for shear cracking in pearlite based on a sequence of a slip developing in the ferrite from a cracked cementite plate and causing cracking in adjacent plates.[12] This mechanism is, of course, related to our model and that of Courtney for the fracture of composites. As applied to deformation and crack formation in pearlite, our model may thus be considered an extension of the Miller and Smith and Courtney mechanisms.[2,12]

Adapting the present model to pearlite deformation and fracture requires a modification to take into account the brittle behavior of the cementite in room temperature tensile deformation. Instead of the previously assumed shear stress criteria for flow, we now assume that cracking and probably macroscopic flow in tension at room temperature occurs when the maximum normal stress σ_{max} in the cementite attains a critical value which is either equal to or directly related to the cementite fracture stress. As before, the stress state in the cementite is a result of a shear stress τ resulting from slip in the α-phase matrix and the applied tensile stress σ. The value of σ_{max} for the configuration shown in Figure 1 is:

$$\sigma_{max} = \frac{\sigma}{2} + \tau_{max}. \tag{7}$$

Unlike the case of a deformable plate-like barrier, the value of τ which is now used to calculate τ_{max} should reflect the total shear strain in the α-phase. Thus, τ_{max} appropriate to the brittle barrier associated with eqn. 6 is given by eqns. 1, 4, and 6, but not eqn. 2.

The critical value of σ_{max} for cementite fracture should be a material constant depending on cementite morphology and dimensions, flaw size, and structure. Since σ_{max} depends on σ as well as τ through eqns. 1, 4, 6, and 7, any increase in τ would decrease the applied stress σ required for the critical value of σ_{max} to cause repeated cementite fracture and flow across a colony. Using flow stress/grain size arguments, or more specifically a carbide cracking mechanism,[10] we see that τ should increase with increasing interlamellar spacing, and thus the yield stress should decrease as interlamellar spacing increases, as has been commonly found.[10,15,16] In addition, since flow in the α phase is required for the yielding of the pearlite colonies (this flow contributing in the form of τ to τ_{max}), the temperature and strain-rate dependence of the yield stress of a pearlitic steel should also be nearly that of the ferrite matrix (assuming the fracture stress of the cementite is independent of temperature and strain rate). This is consistent with the observation that the temperature dependence and strain rate sensitivity of plain carbon steels are relatively independent of composition.[17,18]

Using the proposed model, one can also calculate σ_{max} as a function of the orientation of the cementite lamellae to the stress axis, and this can be related to the geometry of pearlite cracks. Because of the multiplicity of slip in the bcc ferrite, slip should occur initially within the α-phase on a plane and in a direction inclined approximately 45° to the tensile axis. The propagation of this slip band should be controlled by the ability of the slip to repeatedly initiate fracture in the cementite lamellae. Using $\theta_S = \phi_S = 45°$ and eqns. 1, 4, 6, and 7, we find that the maximum value of σ_{max}

for a given ε_p in the α occurs at $\beta_{sa} = 90°$ (i.e., cementite parallel to the tensile axis). On the other hand, if the cementite plates are fixed parallel to the stress axis ($\beta_{sa} = 90°$), one finds the value of σ_{max} at a given ε_p increases as θ_s approaches $0°$ (slip plane is perpendicular to the stress axis). This is simply because the shear strain γ_p and therefore τ become infinitely large for a given axial strain ε_p at $\theta_s = 0$ (see eqn. 4). Because of the Schmid factor at $\theta_s = 0$ ($\cos\theta_s \cos\phi_s = 0$) such a slip system would also require an infinitely large axial stress to activate and is therefore not observed. However, in tensile loading there should be a resultant trend for slip across colonies to occur initially in those colonies where the lamellae are aligned parallel to the stress axis and where the slip plane is inclined at angles $\geqslant 45°$ to stress axis. The tendency to larger angles of inclination occurs so long as slip on planes inclined to the stress axis at $\sim 45°$ (which are activated first) do not fracture the cementite plates.

As will be discussed in detail later,[1] if slip across a colony occurs on a plane of high Schmid factor, other slip systems in the α are not activated, work hardening is small, and slip tends to be inhomogeneous. The combination of inhomogeneous slip in the α coupled with efficient, slip-induced cracking of the cementite should be a dominant factor in the crack formation across pearlite colonies in steel. This combination of inhomogeneous slip/cracking occurs only in those colonies in which a slip plane with a high Schmid factor can efficiently fracture the cementite (and this occurs at $\beta_{sa} = 90°$ and $\theta_s \leqslant 45°$). Thus, it is expected that cracks in pearlite colonies should tend to be inclined at $\sim 45 - 55°$ to the tensile axis in colonies where the lamellae are parallel to the stress axis, as Miller and Smith[12] observe. Furthermore, from this reasoning and eqns. 1 and 7, any increase in σ such as that due to a decrease in test temperature or increase in strain rate, should

result in a smaller value of τ (and therefore less slip) necessary for σ_{max} to cause cementite cracking. Thus, increasingly pronounced microcracking should occur in pearlite at lower temperatures and at higher strain rates, as is also observed.[6,11,19] We recognize that our discussion somewhat oversimplifies the deformation and crack formation in pearlite, but we believe that the basic concepts of the model are a necessary ingredient in any more complete analysis. The deformation of an alloy with a lamellar microstructure in which one of the phases is brittle is currently the subject of a more complete investigation by Stout and Courtney.[20]

SUMMARY

A straightforward model has been presented in which macroscopic yielding of a lamellar alloy is controlled by the ability of a slip system, once activated in the softer phase, to penetrate a plate-like deformable barrier. This "shearing" ability is controlled by the capacity of the given active slip system, together with the applied stress, to generate sufficient shear stress through the thickness of the harder phase to cause macroscopic flow. Results on an individual Widmanstatten colony of an α-β Ti alloy are analyzed to demonstrate the model. Using a maximum normal stress criteria and assuming fracture of the cementite lamellae, we also examine the deformation and crack formation in pearlite in terms of the proposed model.

ACKNOWLEDGEMENTS

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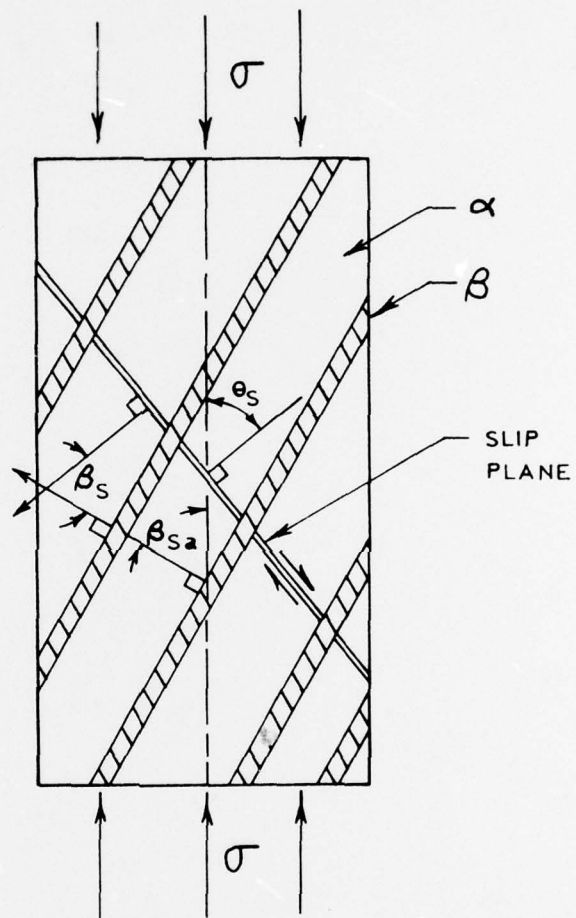20. M. Stout and T. Courtney, unpublished research.

Fig. 1. A schematic diagram illustrating the relationship between
matrix slip and a plate-like barrier in an alloy with a
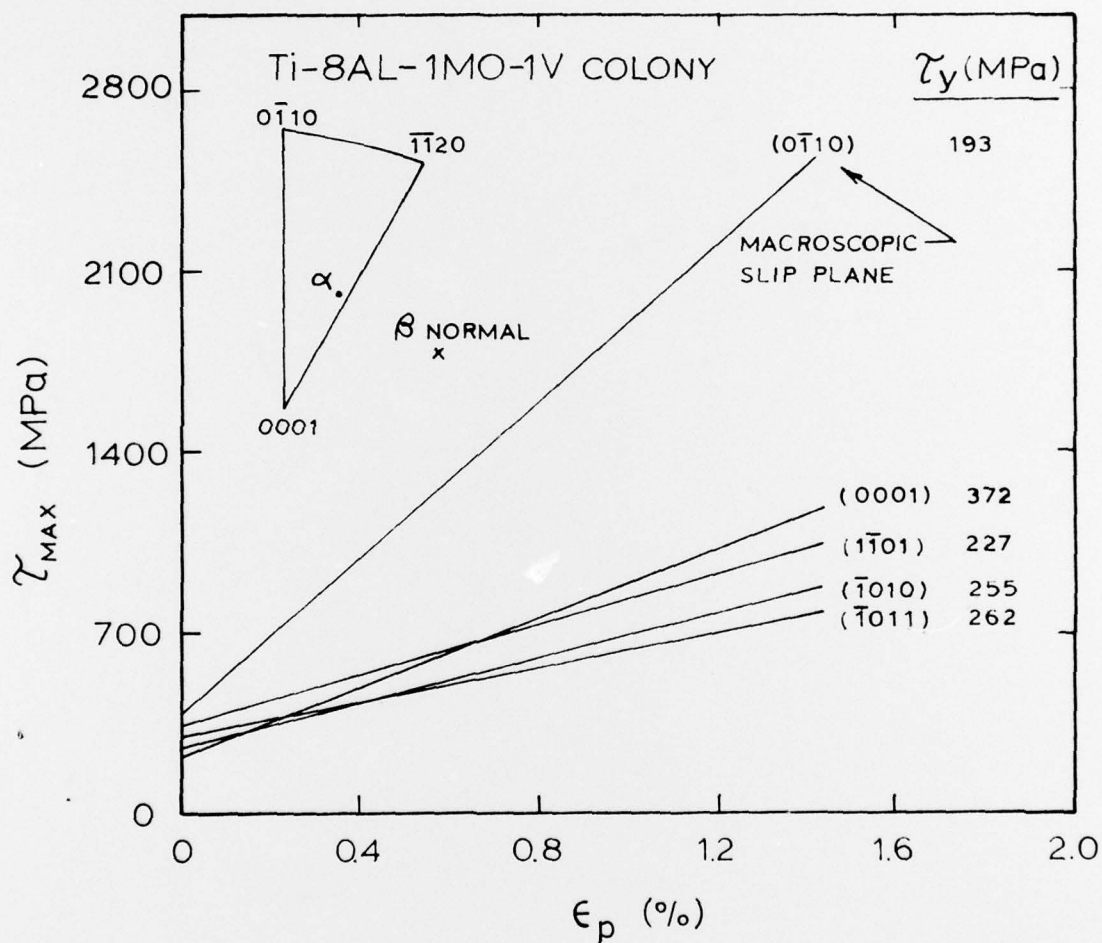lamellar microstructure.

Fig. 2. The influence of the plastic strain ε_p in the α phase on the maximum shear stress τ_{max} in α' for an individual colony of the Ti-8Al-1Mo-1V alloy. The curves were calculated using (in units of MPa): $E = 9.65 \times 10^4$, $E^\alpha = 11.7 \times 10^4$, $\tau^\alpha_{10\bar{1}0} = 205$, $\tau^\alpha_{10\bar{1}1} = 250$, and $\tau^\alpha_{0001} = 275$. The sample was heat treated for two hours at 925°C and quenched.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The flow behavior of an alloy with a two-phase lamellar microstructure is analyzed. Based on the model presented, the macroscopic slip system and the yield strength are controlled by the ability of a slip system, once activated in the softer phase, to shear the plate-like harder phase. This ability to shear is controlled by the capacity of a specific slip system, together with the applied stress, to generate sufficient shear stress through the thickness of the harder phase to cause macroscopic flow. The yield stress (continued)